

Statistics of neutrinos and the double beta decay

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Abstract

We assume that the Pauli exclusion principle is violated for neutrinos, and thus, neutrinos obey at least partly the Bose-Einstein statistics. The parameter $\sin^2 \chi$ is introduced that characterizes the bosonic (symmetric) fraction of the neutrino wave function. Consequences of the violation of the exclusion principle for the two-neutrino double beta decays ($2\nu\beta\beta$ -decays) are considered. This violation strongly changes the rates of the decays and modifies the energy and angular distributions of the emitted electrons. Pure bosonic neutrinos are excluded by the present data. In the case of partly bosonic (or mixed-statistics) neutrinos the analysis of the existing data allows to put the conservative upper bound $\sin^2 \chi < 0.6$. The sensitivity of future measurements of the $2\nu\beta\beta$ -decay to $\sin^2 \chi$ is evaluated.

1 Introduction

Does neutrino respect the exclusion principle of it's inventor? In this paper we assume that Pauli exclusion principle is violated for neutrinos and therefore neutrinos obey (at least partly) the Bose-Einstein statistics. Possible violation of the exclusion principle was discussed in a series of papers [1] though no satisfactory and consistent mechanism of the violation has been proposed so far. The assumption of violation of the Pauli exclusion principle leads to a number of fundamental problems which include loss of a positive definiteness of energy, violation of the CPT invariance, and possibly, of the Lorentz invariance as well as of the unitarity of S-matrix. (For a critical review see ref. [2].) Experimental searches of the effects of the Pauli principle violation for electrons [3] and nucleons [4] have given negative results, leading to extremely strong bounds on the magnitude of violation.

It may happen however that due to unique properties of neutrinos (neutrality, smallness of mass associated to some high mass scales), a violation of the Pauli principle in the neutrino sector is much stronger than in other particle sectors. Therefore one may expect that effects of its violation can be first seen in neutrino physics.

A possibility of the Bose statistics for neutrinos has been first considered in ref. [5] where its effects on the Big Bang Nucleosynthesis (BBN) have been studied. According to [5] the change of neutrino statistics from pure fermionic to pure bosonic diminishes the primordial ^4He abundance by $\sim 4\%$.

The idea of bosonic neutrinos has been proposed independently in ref. [6], where cosmological and astrophysical consequences of this hypothesis have been studied. Bosonic neutrinos might form a cosmological Bose condensate which could account for all (or a part of) the dark matter in the universe. “Wrong” statistics of neutrinos modifies the BBN, leading to the effective number of neutrino species smaller than three. The conclusion in [6] agrees qualitatively with results of [5] though quantitatively a smaller decrease of N_ν is found [7].

As far as the astrophysical consequences are concerned, dynamics of the supernova collapse would be influenced and spectra of the supernova neutrinos may change [6, 8]. The presence of neutrino condensate would enhance contributions of the Z-bursts to the flux of the UHE cosmic rays and lead to substantial refraction effects for neutrinos from remote sources [6].

We assume that the Pauli principle is violated substantially for neutrinos, while the violation is negligible for other particles. In particular, for electrons we will assume the usual Fermi-Dirac (FD) statistics. How to reconcile this pattern of the violation with the fact that in the standard model the left-handed neutrino and electron belong to the same doublet? The answer may be connected to the fact that neutrinos are the only known neutral leptons and thus they can have substantially different properties from those of the charged leptons. In particular, neutrinos can be the Majorana particles and violate lepton number conservation. The difference between charged leptons and neutrinos should be related to breaking of the electro-weak (EW) symmetry, and it can originate from some high mass scale of nature. One may consider scenario where violation of the Pauli principle occurs in a hidden sector of theory related to the Planck scale physics, or strings physics. It could be mediated by some singlets of the Standard model - (heavy) neutral fermions which mix with neutrinos when the EW symmetry is broken. Since only neutrinos can mix with the singlets, effects of the Pauli principle violation would show up first in the neutrino sector and then communicate to other particles. In this way a small or partial violation of the relation between spin and statistics might occur. A violation of the spin-statistics theorem for other particles can be suppressed by an additional power of a small parameter relevant for the violation in the neutrino sector and due to weak coupling of neutrino to other particle sector.

A violation of the Pauli principle for neutrinos should show up in the elementary processes where identical neutrinos are involved. A realistic process for this test is the

two-neutrino double beta decay ($2\nu\beta\beta$ -decay),

$$A \rightarrow A' + 2\bar{\nu} + 2e^- \quad (1)$$

(or similar with neutrinos and positrons). It was shown in [6] that the probability of the decay as well as the energy spectrum and angular distribution of electrons should be affected. Qualitative conclusions were that the pure bosonic neutrino is excluded, whereas large fraction of the bosonic component in a neutrino state is still allowed by the present data. In this connection, a possibility of partly bosonic (mixed-statistics) neutrinos can be considered.

In this paper we perform a detailed study of the effects of bosonic neutrinos on the double beta decay. In sect. 2 we consider the general case of partly bosonic neutrinos. We introduce a phenomenological parameter $\sin^2 \chi$ which describes the fraction of bosonic neutrinos in such a way that a smooth change of $\sin^2 \chi$ from 0 to 1 transforms fermionic neutrinos into bosonic ones. So, in general, neutrinos may possess a kind of mixed or more general statistics than Bose or Fermi ones [9, 10]. In sect. 3 we present an analytic study of the double beta decay probabilities. The exact expressions for the $2\nu\beta\beta$ -decay rates to ground and excited 0^+ and 2^+ states with corresponding nuclear matrix elements (NME's) are given in sect. 4. The results of numerical calculations of the total rates and various distributions for the $2\nu\beta\beta$ -decays of ^{76}Ge and ^{100}Mo are presented in sect 5. In sect. 6. we obtain the bounds on $\sin^2 \chi$ from the existing data and evaluate the sensitivities of future double beta decay experiments. Discussion and conclusions are given in sect. 6.

2 The $2\nu\beta\beta$ -decay for bosonic and partly bosonic neutrinos

In the case of mixed statistics the operator of neutrino state can be written as

$$|\nu\rangle = \hat{a}^+|0\rangle \equiv c_\delta \hat{f}^+|0\rangle + s_\delta \hat{b}^+|0\rangle = c_\delta |f\rangle + s_\delta |b\rangle \quad (2)$$

where $|f\rangle$ and $|b\rangle$ are respectively one particle fermionic and bosonic states. The normalization of $|\nu\rangle$ implies $c_\delta^2 + s_\delta^2 = 1$ ($c_\delta \equiv \cos \delta$ and $s_\delta \equiv \sin \delta$). \hat{f} (\hat{f}^+) and \hat{b} (\hat{b}^+) denote fermionic, and bosonic annihilation (creation) operators.

To develop a formalism for description of identical neutrinos one needs to specify commutation/anti-commutation relations. We assume that they have the following form:

$$\hat{f}\hat{b} = e^{i\phi}\hat{b}\hat{f}, \quad \hat{f}^+\hat{b}^+ = e^{i\phi}\hat{b}^+\hat{f}^+, \quad \hat{f}\hat{b}^+ = e^{-i\phi}\hat{b}^+\hat{f}, \quad \hat{f}^+\hat{b} = e^{-i\phi}\hat{b}\hat{f}^+, \quad (3)$$

where ϕ is an arbitrary phase. Then the two-neutrino state can be defined as

$$|k_1, k_2\rangle = \hat{a}_1^+ \hat{a}_2^+ |0\rangle. \quad (4)$$

For the pure bosonic neutrino one cannot introduce the Majorana mass term. So, the neutrinoless double beta decay should be absent. In the case of partly bosonic neutrino, the neutrino mass would appear due to its fermionic component. This means that the

kinematical mass measured, *e.g.* in the tritium beta decay, would not be the same as the mass found from the neutrinoless beta decay. Such a situation, however, can be realized in the case of the usual fermionic neutrinos too.

The amplitude of the decay of nucleus $A \rightarrow 2\nu + 2e + A'$ can be written as

$$A_{2\beta} = \langle e(p_{e1}), e(p_{e2}), \bar{\nu}(p_{\nu1}), \bar{\nu}(p_{\nu2}), A' | \int d^4x_1 d^4x_2 \psi_\nu(x_1) \psi_\nu(x_2) \mathcal{M}(x_1, x_2) | A \rangle. \quad (5)$$

After making the necessary commutation, according to eq. (3), we obtain

$$A_{2\beta} = A_f [c_\delta^4 + c_\delta^2 s_\delta^2 (1 - \cos \phi)] + A_b [s_\delta^4 + c_\delta^2 s_\delta^2 (1 + \cos \phi)], \quad (6)$$

where A_f and A_b are respectively fermionic (antisymmetric) and bosonic (symmetric) parts of two antineutrino emission. The amplitude can be parametrized as

$$A_{2\beta} = \cos^2 \chi A_f + \sin^2 \chi A_b, \quad (7)$$

where $\cos^2 \chi = c_\delta^4 + c_\delta^2 s_\delta^2 (1 - \cos \phi)$ and $\sin^2 \chi = s_\delta^4 + c_\delta^2 s_\delta^2 (1 + \cos \phi)$.

After integration over the neutrino phase space an interference between fermionic A_f and bosonic A_b parts of the amplitude $A_{2\beta}$ vanishes because the fermionic part is antisymmetric with respect to neutrino interchange, while bosonic is symmetric. The probability of the $2\nu\beta\beta$ -decay is equal to:

$$W_{tot} = \cos^4 \chi W_f + \sin^4 \chi W_b, \quad (8)$$

where $W_{f,b}$ are proportional to $|A_{f,b}|^2$. The expressions for $W_{f,b}$ will be given in the next section.

Qualitative features of the $\beta\beta$ -decay in the presence of the bosonic or partly bosonic neutrinos can be understood using the following consideration. Essentially, the effect of neutrino “bosonization” is that two contributions to the amplitude of the decay from diagrams with permuted neutrino momenta $p_{\nu1} \leftrightarrow p_{\nu2}$ should have relative plus sign instead of minus in the FD-case.

The decay probability, W_b , is proportional to the bilinear combinations of the type $K_m^b K_n^b$, $K_m^b L_n^b$, $L_m^b L_n^b$ (see the next section), where

$$\begin{aligned} K_m^b &\equiv [E_m - E_i + E_{e1} + E_{\nu1}]^{-1} - [E_m - E_i + E_{e2} + E_{\nu2}]^{-1}, \\ L_m^b &\equiv [E_m - E_i + E_{e2} + E_{\nu1}]^{-1} - [E_m - E_i + E_{e1} + E_{\nu2}]^{-1}. \end{aligned} \quad (9)$$

Here E_i is the energy of the initial nuclei, E_m is the energy of the intermediate nuclei, E_{ej} , and $E_{\nu j}$ are the energies of electrons and neutrinos respectively. The factors (9) correspond to the propagators of the intermediate nucleus. The key difference between the bosonic and fermionic cases is the opposite signs of the two terms in the expressions (9). In the case of fermionic neutrinos they enter with the same signs (see, *e.g.* [11]):

$$\begin{aligned} K_m^f &\equiv [E_m - E_i + E_{e1} + E_{\nu1}]^{-1} + [E_m - E_i + E_{e2} + E_{\nu2}]^{-1}, \\ L_m^f &\equiv [E_m - E_i + E_{e2} + E_{\nu1}]^{-1} + [E_m - E_i + E_{e1} + E_{\nu2}]^{-1}. \end{aligned} \quad (10)$$

(Remember that for electrons we assume the normal Fermi statistics.) The terms in (9) correspond to the amplitudes with permuted momenta of both neutrinos and electrons.

In the case of fermionic neutrinos such an interchange flips the sign twice (due to neutrinos and electrons), so that the overall sign turns out to be plus. In the case of bosonic neutrinos the permutation of electrons only changes the sign, and the overall sign is minus.

Experimentally interesting are the $2\nu\beta\beta$ -decays to the ground states $0_{g.s.}^+$ and to excited states 0_1^+ and 2_1^+ . The effect of bosonic neutrinos on the $2\nu\beta\beta$ -decay half-life is different for $J^\pi = 2^+$ and $J^\pi = 0^+$. This can be understood qualitatively, approximating the combinations K_m^b and L_m^b for bosonic neutrinos by

$$K_m^b \approx \frac{E_{e2} - E_{e1} + E_{\nu2} - E_{\nu1}}{(E_m - E_i + E_0/2)^2}, \quad L_m^b \approx \frac{E_{e1} - E_{e2} + E_{\nu2} - E_{\nu1}}{(E_m - E_i + E_0/2)^2}, \quad (11)$$

and the corresponding combinations for the fermionic neutrinos by

$$K_m^f \approx L_m^f \approx \frac{2}{E_m - E_i + E_0/2}. \quad (12)$$

Here $E_0/2 \equiv \langle E_e + E_\nu \rangle$ is the average energy of the leptonic pair, $E_0 \equiv E_i - E_f$ is the energy release in the decay, and E_f is the energy of the final nucleus.

For the $0^+ \rightarrow 0^+$ transitions an appearance of the differences of the electron and neutrino energies in the numerators of (11) leads to substantial (1-3 orders of magnitude) suppression of the total probability. It also modifies the energy distributions of electrons.

The effect of bosonic neutrinos on $0^+ \rightarrow 2^+$ transitions is opposite: The probabilities of transitions are proportional to the combinations $(K_m^b - L_m^b)(K_n^b - L_n^b)$, where

$$(K_m^b - L_m^b) \approx \frac{2(E_{e2} - E_{e1})}{(E_m - E_i + E_0/2)^2}. \quad (13)$$

In the case of fermionic neutrinos the combination $(K_m^f - L_m^f)$ has an additional factor $(E_{\nu2} - E_{\nu1})/(E_m - E_i + E_0/2)$ and the suppression is stronger. Parametrically the probabilities of the $0^+ \rightarrow 2^+$ and $0^+ \rightarrow 0^+$ transitions become of the same order for bosonic neutrinos.

In the decay rates, the kinematical factors $K_m^{f,b}$ and $L_n^{f,b}$ are weighted with the corresponding nuclear matrix elements (NME's). Let us introduce the ratio

$$r_0(J^\pi) \equiv \frac{W_b(J^\pi)}{W_f(J^\pi)}, \quad (14)$$

of the decay probabilities to ground ($J^\pi = 0_{g.s.}^+$) and excited ($J^\pi = 0_1^+, 2_1^+$) states in pure bosonic $W_b(J^\pi)$ and pure fermionic cases $W_f(J^\pi)$. In general, to find $r_0(J^\pi)$ one needs to calculate the NME for a given transition within an appropriate nuclear model. The situation is simplified for those nuclear systems, where the transition via solely the ground state of the intermediate nuclei $m = 1$ dominates [12, 13, 14]. For those nuclei the single state dominance (SSD) approximation (hypothesis) can be used. In this case the NME's can be factored out in the rates and therefore cancel in the ratio $r_0(J^\pi)$.

Let us consider the characteristics of the $\beta\beta$ decay to the ground and excited states J^π in the mixed-statistic case of partly bosonic neutrinos. According to our considerations

the total decay probability and the normalized total differential rate can be written as

$$W_{tot}(J^\pi) = \cos^4 \chi W_f(J^\pi) + \sin^4 \chi W_b(J^\pi), \quad (15)$$

$$P(J^\pi) = \frac{dW_{tot}(J^\pi)}{W_{tot}(J^\pi)} = \frac{\cos^4 \chi d\omega_f(J^\pi) + \sin^4 \chi r_0(J^\pi) d\omega_b(J^\pi)}{\cos^4 \chi + \sin^4 \chi r_0(J^\pi)}, \quad (16)$$

where

$$d\omega_f(J^\pi) \equiv \frac{dW_f(J^\pi)}{W_f(J^\pi)}, \quad d\omega_b(J^\pi) \equiv \frac{dW_b(J^\pi)}{W_b(J^\pi)} \quad (17)$$

are the normalized distributions. Here $dW_f(J^\pi)$ and $dW_b(J^\pi)$ are the differential rates of the $2\nu\beta\beta$ -decay for the pure fermionic and bosonic neutrinos. In the case of single state dominance due to factorization, the normalized distributions do not depend on the uncertainties of the matrix elements [13, 14]. In general, the factorization does not occur and the uncertainties of nuclear matrix elements restrict substantially the sensitivity of the $\beta\beta$ -decay to statistics of neutrinos.

3 Rates and nuclear matrix elements

For the cases of pure fermionic and bosonic neutrinos we outline the derivation of $2\nu\beta\beta$ -decay rates. The relevant nuclear matrix elements will be evaluated and discussed using the SSD and HSD (higher states dominance) hypothesis [13, 14].

The matrix element of the $2\nu\beta\beta$ -decay process takes the form

$$\begin{aligned} & \langle f | S^{(2)} | i \rangle = \\ & \frac{(-i)^2}{2} \int \langle e(p_{e1}), e(p_{e2}), \bar{\nu}(p_{\nu1}), \bar{\nu}(p_{\nu2}), A' | T [\mathcal{H}^\beta(x_1) \mathcal{H}^\beta(x_2)] | A \rangle dx_1 dx_2, \end{aligned} \quad (18)$$

where the weak β -decay Hamiltonian is

$$\mathcal{H}^\beta(x) = \frac{G_F}{\sqrt{2}} [\bar{e}(x) \gamma^\mu (1 + \gamma_5) \nu_e(x)] J_\mu(x) + h.c.. \quad (19)$$

Here, $J_\mu(x)$ is the weak charged (nuclear) hadron current in the Heisenberg representation. The T -product of the two hadron currents can be written as

$$\begin{aligned} & T(H^\beta(x_1) H^\beta(x_2)) = \\ & \Theta(x_{10} - x_{20}) H^\beta(x_1) H^\beta(x_2) + \Theta(x_{20} - x_{10}) H^\beta(x_2) H^\beta(x_1). \end{aligned} \quad (20)$$

In the derivation of the $2\nu\beta\beta$ -decay rate a number of conventional approximations have been used: i) Only the $s_{1/2}$ wave states of the outgoing leptons are taken into account. ii) The contribution of the double Fermi matrix element to the decay rate is neglected as the initial and final nuclei belong to different isospin multiplets. iii) Only the leading order ($1/m_p$) Gamow-Teller operators in the non-relativistic reduction of the hadron current are retained.

For the differential $2\nu\beta\beta$ -decay rates to 0^+ ground state and 2^+ excited state we obtain

$$dW_{f,b}(J^+) = a_{2\nu} F(Z_f, E_{e1}) F(Z_f, E_{e2}) \mathcal{M}_{J^\pi}^{f,b} d\Omega, \quad (21)$$

where $a_{2\nu} = (G_\beta^4 g_A^4 m_e^9)/(64\pi^7)$ and $G_\beta = G_F \cos \theta_c$ (G_F is Fermi constant, θ_c is Cabibbo angle). $F(Z_f, E_e)$ denotes the relativistic Coulomb factor and g_A is the axial-vector coupling constant. The upper index f (b) stands for fermionic (bosonic) neutrinos.

The phase space factor equals

$$d\Omega = \frac{1}{m_e^{11}} E_{e1} p_{e1} E_{e2} p_{e2} E_{\nu 1}^2 E_{\nu 2}^2 \delta(E_{e1} + E_{e2} + E_{\nu 1} + E_{\nu 2} + E_f - E_i) \times \\ dE_{e1} dE_{e2} dE_{\nu 1} dE_{\nu 2} d\cos\theta. \quad (22)$$

Here, θ is the angle between the outgoing electrons. $\mathcal{M}_{J^\pi}^{f,b}$ ($J^\pi = 0^+, 2^+$) consists of the products of nuclear matrix elements:

$$\begin{aligned} \mathcal{M}_{0^+}^{f,b} &= \frac{m_e^2}{4} \left[|\mathcal{K}_{0^+}^{f,b} + \mathcal{L}_{0^+}^{f,b}|^2 + \frac{1}{3} |\mathcal{K}_{0^+}^{f,b} - \mathcal{L}_{0^+}^{f,b}|^2 \right] \\ &\quad - \frac{m_e^2}{4} \left[|\mathcal{K}_{0^+}^{f,b} + \mathcal{L}_{0^+}^{f,b}|^2 - \frac{1}{9} |\mathcal{K}_{0^+}^{f,b} - \mathcal{L}_{0^+}^{f,b}|^2 \right] \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1} E_{e2}}, \\ \mathcal{M}_{2^+}^{f,b} &= m_e^2 |\mathcal{K}_{2^+}^{f,b} - \mathcal{L}_{2^+}^{f,b}|^2 \left(1 + \frac{1}{3} \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1} E_{e2}} \right) \end{aligned} \quad (23)$$

with

$$\begin{aligned} \mathcal{K}_{J^+}^{f,b} &= \frac{m_e}{\sqrt{s}} \sum_m \langle J_f^\pi || \sum_j \tau_j^+ \sigma_j || 1_m^+ \rangle \langle 1_m^+ || \sum_k \tau_k^+ \sigma_k || 0_i^+ \rangle K_m^{f,b} \\ \mathcal{L}_{J^+}^{f,b} &= \frac{m_e}{\sqrt{s}} \sum_m \langle J_f^\pi || \sum_j \tau_j^+ \sigma_j || 1_m^+ \rangle \langle 1_m^+ || \sum_k \tau_k^+ \sigma_k || 0_i^+ \rangle L_m^{f,b}. \end{aligned} \quad (24)$$

Here, $s = 1$ for $J = 0$ and $s = 3$ for $J = 2$. $|0_i^+ \rangle$, $|0_f^+ \rangle$ ($|2_f^+ \rangle$) and $|1_m^+ \rangle$ are, respectively, the states of the initial, final and intermediate nuclei with corresponding energies E_i , E_f and E_m . The energy denominators $K_m^{f,b}$ and $L_m^{f,b}$ were introduced in Eqs. (9) and (10).

3.1 Higher states dominance

The $2\nu\beta\beta$ -decay rates are usually evaluated in the approximation in which the sum of the two lepton energies in the denominator of the nuclear matrix element is replaced with their average value $E_0/2$

$$E_m - E_i + E_{ej} + E_{\nu k} \approx E_m - E_i + E_0/2 \quad (25)$$

($j, k = 1, 2$). The main purpose of this approximation is to factorize the lepton and nuclear parts in the calculation of the $2\nu\beta\beta$ -decay half-life. This approximation is justified if the transitions through the higher-lying states of the intermediate nucleus (at least few MeV above the ground state of (A,Z+1) nucleus) give the dominant contribution to the

$2\nu\beta\beta$ -decay amplitude. This assumption is called the higher states dominance (HSD) hypothesis. It is expected to be realized for A= 48, 76, 82, 130, 136 nuclear systems.

Assuming the HSD hypothesis we obtain for fermionic neutrinos

$$\begin{aligned}\mathcal{M}_{0+}^f &\simeq |M_{GT}^{(1)}(0^+)|^2 \left(1 - \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1}E_{e2}}\right), \\ \mathcal{M}_{2+}^f &= |M_{GT}^{(3)}(2^+)|^2 \frac{(E_{e1} - E_{e2})^2 (E_{\nu1} - E_{\nu2})^2}{2m_e^6} \left(1 + \frac{1}{3} \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1}E_{e2}}\right).\end{aligned}\quad (26)$$

In the case of bosonic neutrinos we end up with

$$\begin{aligned}\mathcal{M}_{0+}^b &= |M_{GT}^{(2)}(0^+)|^2 \left[\frac{3(E_{\nu2} - E_{\nu1})^2 + (E_{e2} - E_{e1})^2}{48m_e^2} - \frac{9(E_{\nu2} - E_{\nu1})^2 - (E_{e2} - E_{e1})^2}{144m_e^2} \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1}E_{e2}} \right], \\ \mathcal{M}_{2+}^b &= |M_{GT}^{(2)}(2^+)|^2 \frac{(E_{e1} - E_{e2})^2}{4m_e^2} \left(1 + \frac{1}{3} \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1}E_{e2}}\right).\end{aligned}\quad (27)$$

The Gamow-Teller matrix elements are given by

$$M_{GT}^{(r)}(J^\pi) = \frac{(2m_e)^r}{\sqrt{s}} \sum_m \frac{\langle J_f^\pi || \sum_j \tau_j^+ \sigma_j || 1_m^+ \rangle \langle 1_m^+ || \sum_k \tau_k^+ \sigma_k || 0_i^+ \rangle}{(E_m - E_i + E_0/2)^r} \quad (28)$$

($r = 1, 2, 3$).

The full decay probabilities in pure bosonic W_b and pure fermionic W_f cases can be written as

$$\begin{aligned}W_f(0^+) &= |M_{GT}^{(1)}(0^+)|^2 \mathcal{I}_{HSD}^f(0^+), \\ W_f(2^+) &= |M_{GT}^{(3)}(2^+)|^2 \mathcal{I}_{HSD}^f(2^+)\end{aligned}\quad (29)$$

and

$$W_b(J^\pi) = |M_{GT}^{(2)}(J^\pi)|^2 \mathcal{I}_{HSD}^f(J^\pi), \quad (30)$$

where the phase space integrals are given by

$$\begin{aligned}\mathcal{I}_{HSD}^{f,b}(J^\pi) &= \frac{2a_{2\nu}}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} f_{J^\pi}^{f,b}(E_{e1}, E_{e2}, E_{\nu1}, E_{\nu2}) F_0(Z_f, E_{e1}) p_{e1} E_{e1} dE_{e1} \times \\ &\quad \int_{m_e}^{E_i - E_f - E_{e1}} F_0(Z_f, p_{e2}) p_{e2} E_{e2} dE_{e2} \int_0^{E_i - E_f - E_{e1} - E_{e2}} E_{\nu2}^2 E_{\nu1}^2 dE_{\nu1}\end{aligned}\quad (31)$$

with $E_{\nu2} = E_i - E_f - E_{e1} - E_{e2} - E_{\nu1}$ and

$$\begin{aligned}f_{J^\pi}^f(E_{e1}, E_{e2}, E_{\nu1}, E_{\nu2}) &= 1 & (J^\pi = 0^+), \\ &= \frac{(E_{e1} - E_{e2})^2 (E_{\nu1} - E_{\nu2})^2}{2m_e^6} & (J^\pi = 2^+), \\ f_{J^\pi}^b(E_{e1}, E_{e2}, E_{\nu1}, E_{\nu2}) &= \frac{3(E_{\nu2} - E_{\nu1})^2 + (E_{e2} - E_{e1})^2}{48m_e^2} & (J^\pi = 0^+), \\ &= \frac{(E_{e1} - E_{e2})^2}{4m_e^2} & (J^\pi = 2^+).\end{aligned}\quad (32)$$

The $2\nu\beta\beta$ -decay half-life is

$$T_{1/2}^{f,b}(J^\pi) = \frac{\ln 2}{W_{f,b}(J^\pi)}. \quad (33)$$

3.2 Single state dominance

The single state dominance hypothesis assumes that the $2\nu\beta\beta$ -decays with 1^+ ground state of the intermediate nucleus (e.g., A=100, 116 and 128 nuclear systems) are only governed by the two virtual β -transitions: i) the first one connects the ground state of the initial nucleus with 1_1^+ intermediate state; ii) the second one proceeds from 1_1^+ state to the final ground state. In this case we find

$$\begin{aligned} \mathcal{M}_{0^+}^{f,b} &= |M_{g.s.}(0^+)|^2 m_e^2 \left[\frac{1}{3} (K^{f,b} K^{f,b} + L^{f,b} L^{f,b} + K^{f,b} L^{f,b}) - \right. \\ &\quad \left. \frac{1}{9} (2K^{f,b} K^{f,b} + 2L^{f,b} L^{f,b} + 5K^{f,b} L^{f,b}) \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1} E_{e2}} \right], \\ \mathcal{M}_{2^+}^{f,b} &= m_e^2 |M_{g.s.}(2^+)|^2 (K^{f,b} - L^{f,b})^2 \left(1 + \frac{1}{3} \frac{\vec{p}_{e1} \cdot \vec{p}_{e2}}{E_{e1} E_{e2}} \right) \end{aligned} \quad (34)$$

with $K^{f,b} \equiv K_{m=1}^{f,b}$, $L^{f,b} \equiv L_{m=1}^{f,b}$ and

$$M_{g.s.}(J^\pi) = \frac{1}{\sqrt{s}} \langle J_f^\pi || \sum_j \tau_j^+ \sigma_j || 1_1^+ \rangle \langle 1_1^+ || \sum_k \tau_k^+ \sigma_k || 0_i^+ \rangle. \quad (35)$$

The value of the matrix element $M_{g.s.}(J^\pi)$ can be determined in a model independent way from the single β -decay and electron capture measurements. From the experimental values of $\log ft$ ¹ for the electron capture and the single β decay of the ground state of the intermediate nucleus with $J^\pi = 1^+$ we obtain

$$\begin{aligned} | \langle 1_1^+ || \sum_k \tau_k^+ \sigma_k || 0_i^+ \rangle | &= \frac{1}{g_A} \sqrt{\frac{3D}{ft_{EC}}}, \\ | \langle J_f^\pi || \sum_j \tau_j^+ \sigma_j || 1_1^+ \rangle | &= \frac{1}{g_A} \sqrt{\frac{3D}{ft_{\beta^-}}}. \end{aligned} \quad (36)$$

Here $D = G_\beta^4 g_A^4 / (8\pi^7)$.

Within the SSD approach for the full decay probabilities we find

$$W_{f,b}(J^\pi) = |M_{g.s.}(J^\pi)|^2 \mathcal{I}_{SSD}^{f,b}(J^\pi), \quad (37)$$

where

$$\begin{aligned} \mathcal{I}_{SSD}^{f,b}(J^\pi) &= \frac{2a_{2\nu}}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} g_{J^\pi}^{f,b}(E_{e1}, E_{e2}, E_{\nu1}, E_{\nu2}) F_0(Z_f, E_{e1}) p_{e1} E_{e1} dE_{e1} \times \\ &\quad \int_{m_e}^{E_i - E_f - E_{e1}} F_0(Z_f, p_{e2}) p_{e2} E_{e2} dE_{e2} \int_0^{E_i - E_f - E_{e1} - E_{e2}} E_{\nu2}^2 E_{\nu1}^2 dE_{\nu1} \end{aligned} \quad (38)$$

¹Because of wide range of β -lifetimes, transitions are classified by $\log_{10} ft$ values (see e.g. [15]). t and f denote the measured half-life and the Fermi integral, respectively.

with

$$\begin{aligned} g_{0+}^{f,b}(E_{e1}, E_{e2}, E_{\nu1}, E_{\nu2}) &= m_e^2 \left[\frac{1}{3} (K^{f,b} K^{f,b} + L^{f,b} L^{f,b} + K^{f,b} L^{f,b}) \right] \\ g_{2+}^{f,b}(E_{e1}, E_{e2}, E_{\nu1}, E_{\nu2}) &= m_e^2 (K^{f,b} - L^{f,b})^2. \end{aligned} \quad (39)$$

4 Characteristics of double beta decays

In what follows we calculate the characteristics of the double beta decay mainly for two nuclei ^{100}Mo and ^{76}Ge for which the highest number of events has been collected in experiment (see Ref. [16] and [17] respectively).

4.1 Double beta decay of ^{100}Mo

The NEMO-3 collaboration has detected about 219 000 ($0^+ \rightarrow 0^+$)-decays of ^{100}Mo [16]. The signal to background ratio is very high $S/B = 44$ and the background is at the level of 2.5% only. All parameters of the decay: the sum of the electron energies, the energy of each electron and the angular distribution (angular correlation of electrons) have been measured.

In the case of ^{100}Mo the decay proceeds mainly through the 1^+ intermediate nucleus and the single state dominance (SSD) hypothesis should give a good approximation. This is also confirmed by spectra measurements in NEMO-3 experiment [18, 19]. Since $E_m - E_i \sim E_i - E_f$, the lepton energies are important in the energy-denominators (9), and consequently, in the rates.

In the SSD approximation one can calculate the probability (NME) using existing experimental data for the beta-decay and the electron capture of ^{100}Tc which is the intermediate dominating state. Accuracy of this “phenomenological” calculation is about 50%, mainly because of poor experimental accuracy for the electron capture process.

Using the SSD approximation we calculated the $2\nu\beta\beta$ -decay half-life of ^{100}Mo to ground state for fermionic [14] and bosonic neutrinos (see sect. 3)

$$T_{1/2}^f(0_{g.s.}^+) = 6.8 \cdot 10^{18} \text{years}, \quad T_{1/2}^b(0_{g.s.}^+) = 8.9 \cdot 10^{19} \text{years}, \quad (40)$$

so that the ratio of probabilities equals

$$r_0(0_{g.s.}^+) = 0.076. \quad (41)$$

The ratio $r_0(0_{g.s.}^+)$ determines the weight with which the bosonic component enters the total rate and differential distribution [see Eq.(15)]. For small r_0 , a substantial modification of the distribution is expected for $\sin^2 \chi$ being close to 1.

The higher intermediate levels can give some (basically unknown) contribution and this produces a systematic error in our analysis. To evaluate effect of the higher states, one can consider the extreme case described by the higher states dominance (HSD) approximation, which allows one to factorize the nuclear matrix element and integration over the phase space of outgoing leptons. In this case the main contribution to the $2\nu\beta\beta$ -decay matrix element comes from the transition through higher energy states (including the region

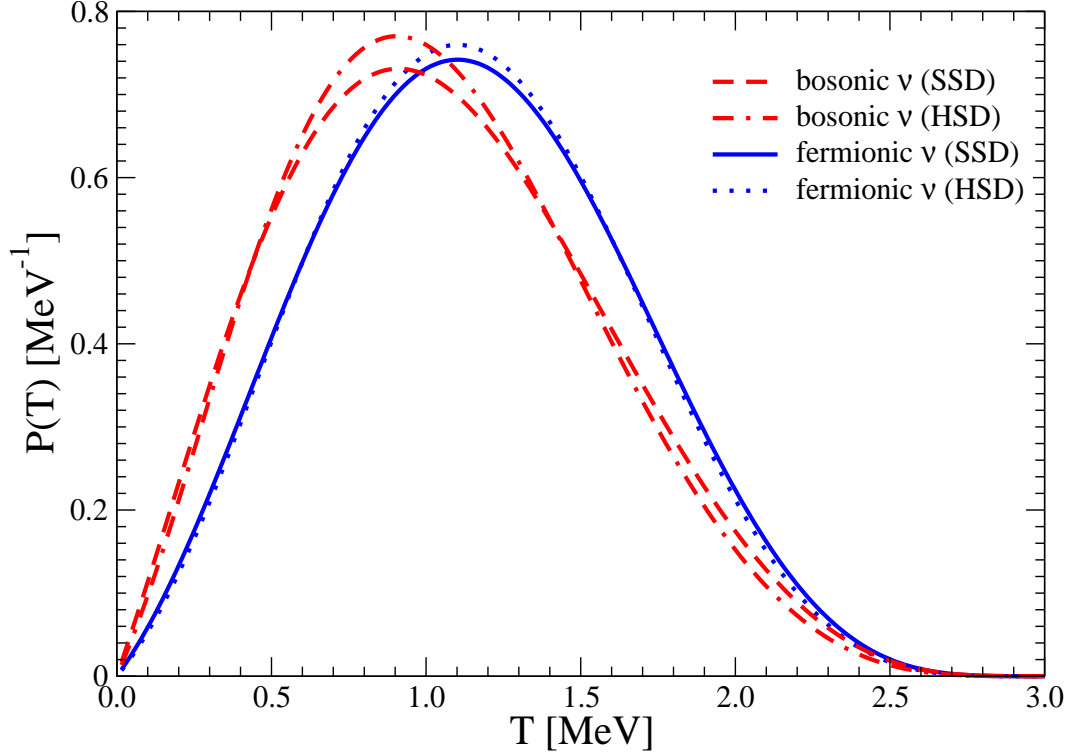


Figure 1: The differential decay rates normalized to the total decay rate vs. the sum of the kinetic energy of outgoing electrons T for $2\nu\beta\beta$ -decay of ^{100}Mo to the ground state of final nucleus. The results are presented for the cases of pure fermionic and pure bosonic neutrinos. The calculations have been performed within the single-state dominance hypothesis (SSD) and with the assumption of dominance of higher lying states (HSD).

of the Gamow-Teller resonance) of the intermediate nucleus. Thus, the lepton energies in the denominators (9) can be neglected (or approximated by $(E_f - E_i)/2$) due to a large value of $E_n - E_i$. The fermionic and bosonic $2\nu\beta\beta$ -decay rates are associated with different nuclear matrix elements [see Eq. (26) and (27)]. They can be evaluated within an appropriate nuclear model like Quasiparticle Random Phase approximation (QRPA) or Nuclear Shell Model (NSM). Then, the evaluated values of $2\nu\beta\beta$ -decay half-life and ratio $r_0(0_{g.s.}^+)$ are model dependent. Contrary, the normalized differential characteristics are model independent for cases of pure fermionic and bosonic neutrinos.

The energy spectra of electrons calculated in the SSD and HSD approximations are presented in the figs. (1) and (2). The SSD approximation gives slightly wider spectra of two electrons both for fermionic and bosonic neutrinos. The spectra for bosonic neutrinos are softer in both approximations. In particular, the maxima of SSD and HSD spectra are shifted to low energies for bosonic neutrinos by about 15 % with respect to fermionic-neutrino spectra. This shift does not depend on the approximation and therefore can be considered as the solid signature of bosonic neutrino. Also the energy spectrum for single electron becomes softer in the bosonic case (Fig. 2).

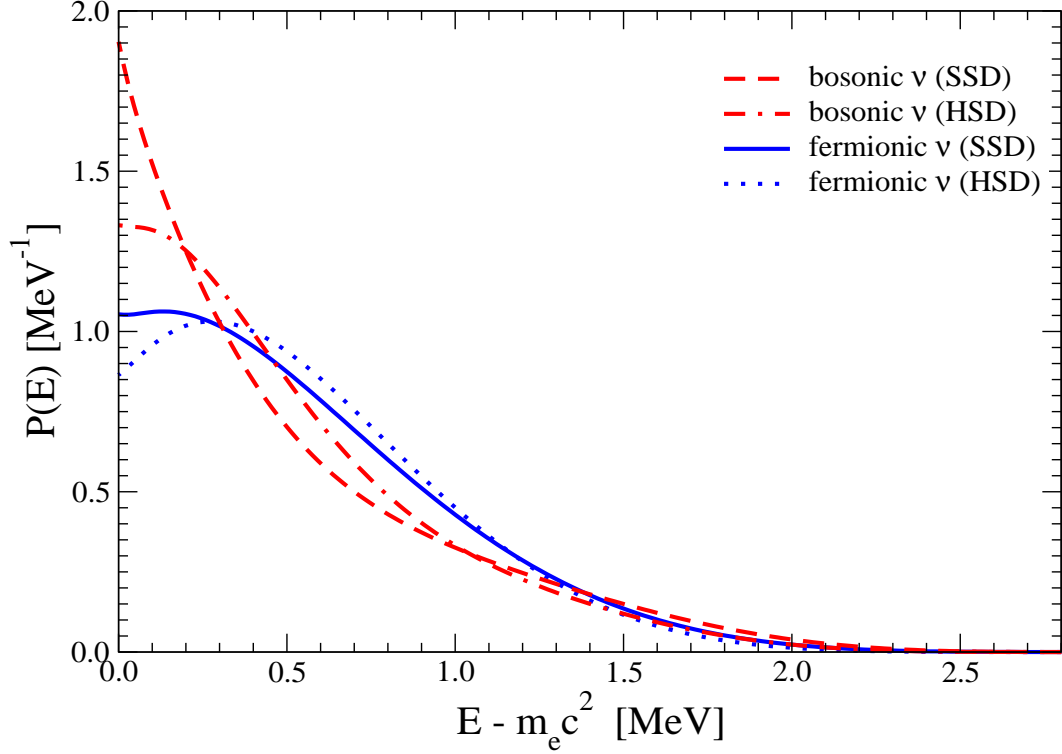


Figure 2: The single electron differential decay rate normalized to the total decay rate vs. the electron energy for $2\nu\beta\beta$ -decay of ^{100}Mo to the ground state of final nucleus. E and m_e represent the energy and mass of the electron, respectively. The results are presented for the cases of pure fermionic and pure bosonic neutrinos. The conventions are the same as in Fig. 1.

In Fig. 3 we show the energy spectra of two electrons for different values of the bosonic-fraction $\sin^2 \chi$. With increase of $\sin^2 \chi$ the spectra shift to smaller energies. Due to smallness of r_0 substantial shift occurs only when $\sin^2 \chi$ is close to 1.0

In Fig. 4 we show the energy spectra of single electrons for different values of $\sin^2 \chi$. A substantial change occurs at very low energies, with $E_{kin} = 0.3$ MeV being a fixed point. For $E_{kin} < 0.3$ MeV the distribution increase with $\sin^2 \chi$, whereas for $E_{kin} = 0.3 - 1.4$ MeV it decreases.

As we mentioned before, the rates of transitions to first excited 2_1^+ state are affected by the presence of bosonic neutrino component in the opposite (to 0^+) way. Furthermore, in the SSD approximation the ratio of decay rates to the excited 2^+ state and to the $0_{g.s.}^+$ ground state does not depend on the $\log ft_{EC}$ value, which is not measured accurately enough. For the $2\nu\beta\beta$ -decay of ^{100}Mo within the SSD approximation we obtain

$$\begin{aligned} T_{1/2}(2_1^+) &= 1.7 \cdot 10^{23} \text{ years} && (\text{fermionic } \nu) \\ &= 2.4 \cdot 10^{22} \text{ years} && (\text{bosonic } \nu). \end{aligned} \quad (42)$$

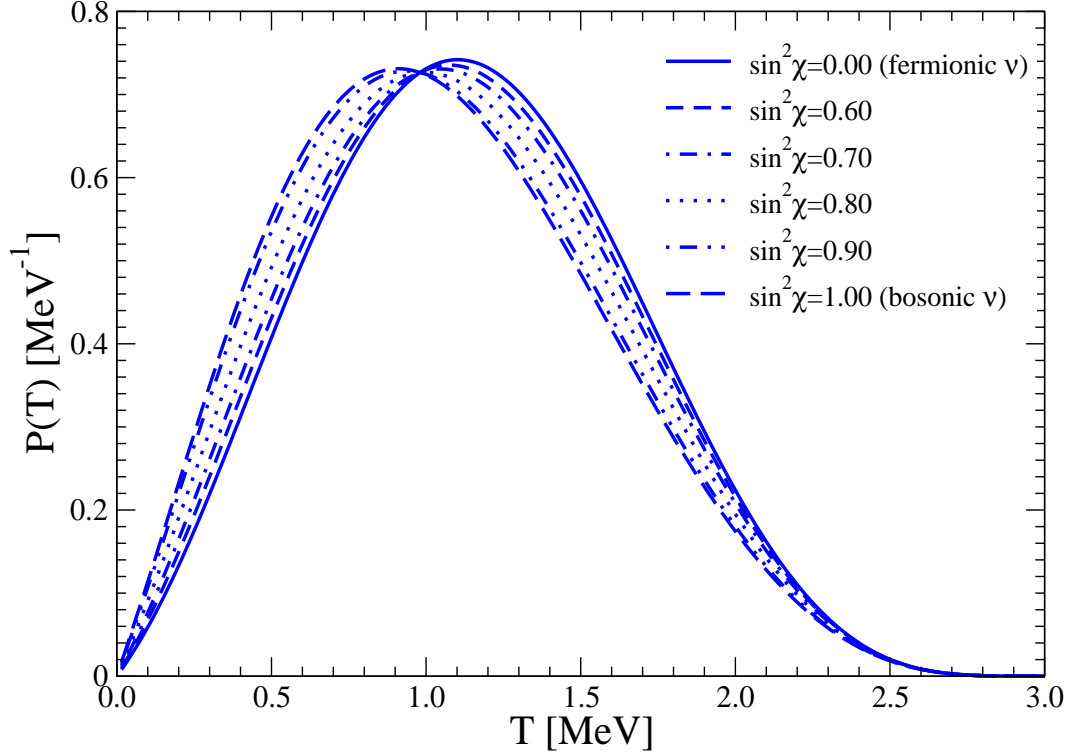


Figure 3: The differential decay rates normalized to the total decay rate vs. the sum of the kinetic energy of outgoing electrons T for $2\nu\beta\beta$ -decay of ^{100}Mo to the ground state of final nucleus. The results are presented for different values of the squared admixture $\sin^2\chi$ of the bosonic component. The spectra have been calculated in the SSD approximation.

Then the ratio of the bosonic and fermionic half-lives equals

$$r_0(2_1^+) = 7.1. \quad (43)$$

The bosonic rate is larger in agreement with our qualitative consideration in sect. 2.

The best lower bound on the $2\nu\beta\beta$ -decay half-life to excited 2_1^+ state is $1.6 \cdot 10^{21}$ years [20]. The current limit of NEMO-3 experiment is $1.1 \cdot 10^{21}$ years [21] (for 1 year of measurements). After 5 years of measurements with the present low-radon background conditions sensitivity will increase up to $\sim 10^{22}$ years thus approaching the prediction in the case of bosonic neutrinos. Due to the large value of r_0 even a small fraction of bosonic neutrinos can produce significant distortion of the standard (fermionic) spectra.

Modifications of the spectra are opposite for the decay of ^{100}Mo into 2^+ excited state: the spectra become harder with increase of $\sin^2\chi$ (see Fig. 5 and 6). This is apparently related to the change of the spin of the nuclei. In the case of $0^+ - 2_1^+$ transition the leptonic system should take spin 2 and therefore due to polarization of leptons (determined by V - A character of interactions) both electrons move preferably in the same direction (hemisphere) and two antineutrinos in the opposite direction with the corresponding Pauli blocking factor. In the case of bosonic neutrinos the Pauli blocking effect is reduced and therefore the electrons can be more aligned and consequently have higher energies.

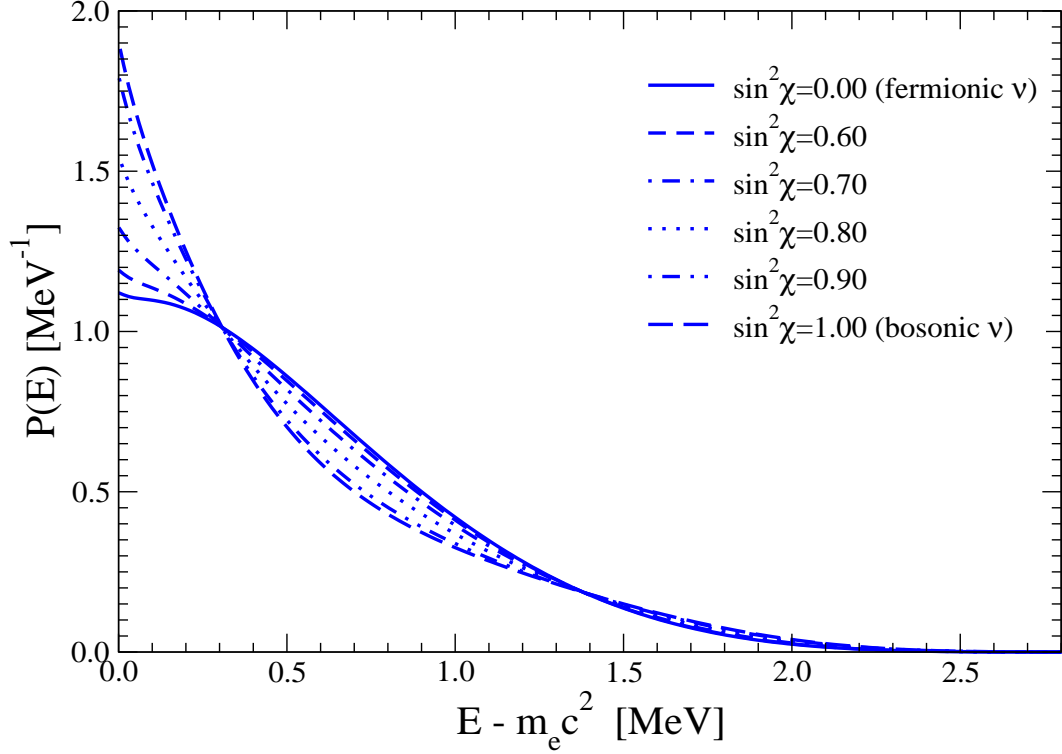


Figure 4: The single electron differential decay rate normalized to the total decay rate vs. the electron energy for $2\nu\beta\beta$ -decay of ^{100}Mo to the ground state of final nucleus. The results are presented for different values of the squared admixture $\sin^2\chi$ of the bosonic component. The spectra have been calculated in the SSD approximation. The conventions are the same as in Fig. 2.

Correspondingly the spectrum becomes harder. In the case of $0^+ - 0^+$ transition the total leptonic momentum is zero, so that the electrons move in the opposite directions.

According to Fig. 5 even 10 % of "bosonic" admixture gives substantial distortion effect and this fact can be used in the future experiments.

The angular distribution of outgoing electrons [13] can be written as

$$\frac{dW_{f,b}(J^\pi)}{d\cos\theta} = \frac{W_{f,b}(J^\pi)}{2}(1 + \kappa^{f,b}(J^\pi)\cos\theta), \quad (44)$$

where θ is the angle between two electrons. For $0^+ - 0^+$ transition and fermionic neutrinos in the SSD approximation

$$\kappa^f(0_{g.s.}^+) = -0.627 \quad (\text{fermionic neutrino}). \quad (45)$$

(The HSD approximation gives similar number: -0.646 .) Notice that the preferable direction is $\theta = 180^\circ$ when electrons move in the opposite directions. The configuration with the same direction of two electrons is suppressed. For bosonic neutrinos we find

$$\kappa^b(0_{g.s.}^+) = -0.344 \quad (\text{bosonic neutrino}). \quad (46)$$

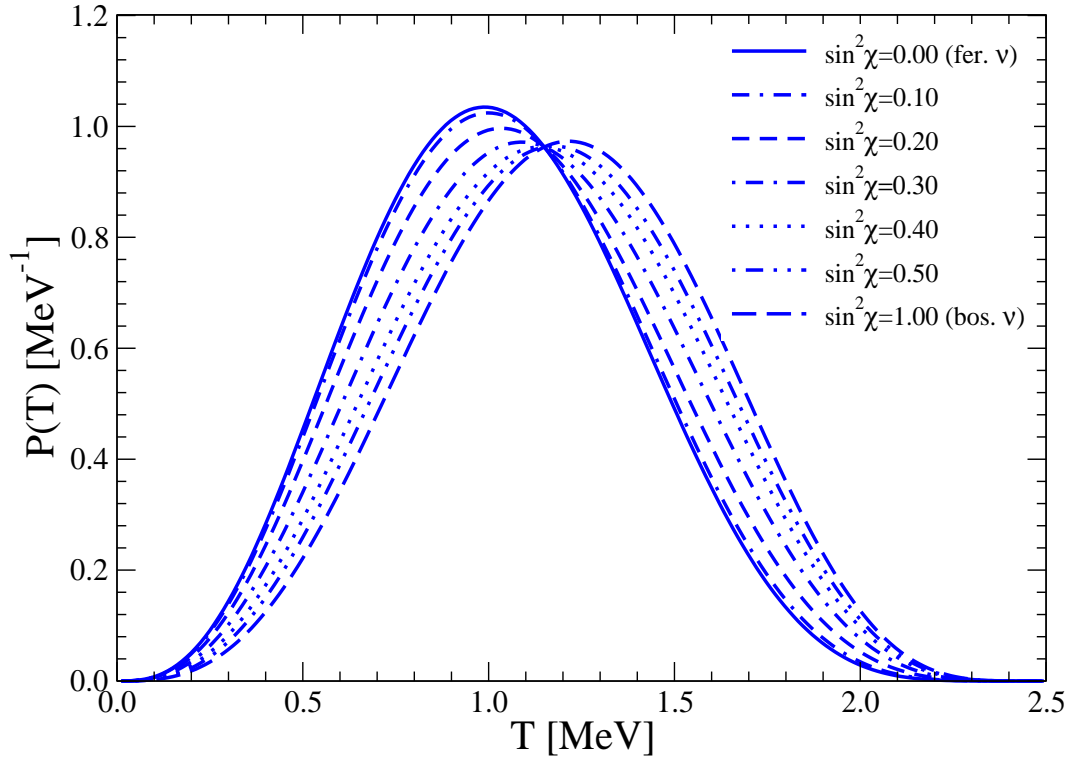


Figure 5: The differential decay rates normalized to the total decay rate vs. the sum of the kinetic energy of outgoing electrons T for $2\nu\beta\beta$ -decay of ^{100}Mo to the excited 2_1^+ state of final nucleus. The results are presented for different values of the squared admixture $\sin^2\chi$ of the bosonic component. The spectra have been calculated in the SSD approximation.

(The HSD approximation gives -0.422 .) So, the configuration with the same direction of electrons is less suppressed and the distribution is more isotropic (flatter) than in the fermionic case.

4.2 ^{76}Ge double beta decay

The statistics of ^{76}Ge decays is about 113000 events, the background is rather high, $S/B = 1.3$, and only the sum of two electron energies is measured [17]. The systematic error can be as large as 10% and the main source of the error is the background. One has to estimate this background independently and make subtraction. So, one can shift the spectrum and its maximum within the error. Furthermore, the energy spectrum of two electrons starts to dominate over the background above 0.7 MeV which means that the maximum of the spectrum is not observed. The advantage of ^{76}Ge is that there is practically no difference between the results of HSD and SSD approximations for the energy distributions because the nearest 1_1^+ state of the intermediate nucleus is lying high enough. Thus, one does not need to make assumptions about SSD or HSD. In this way the conclusion does not depend on the nuclear structure details.

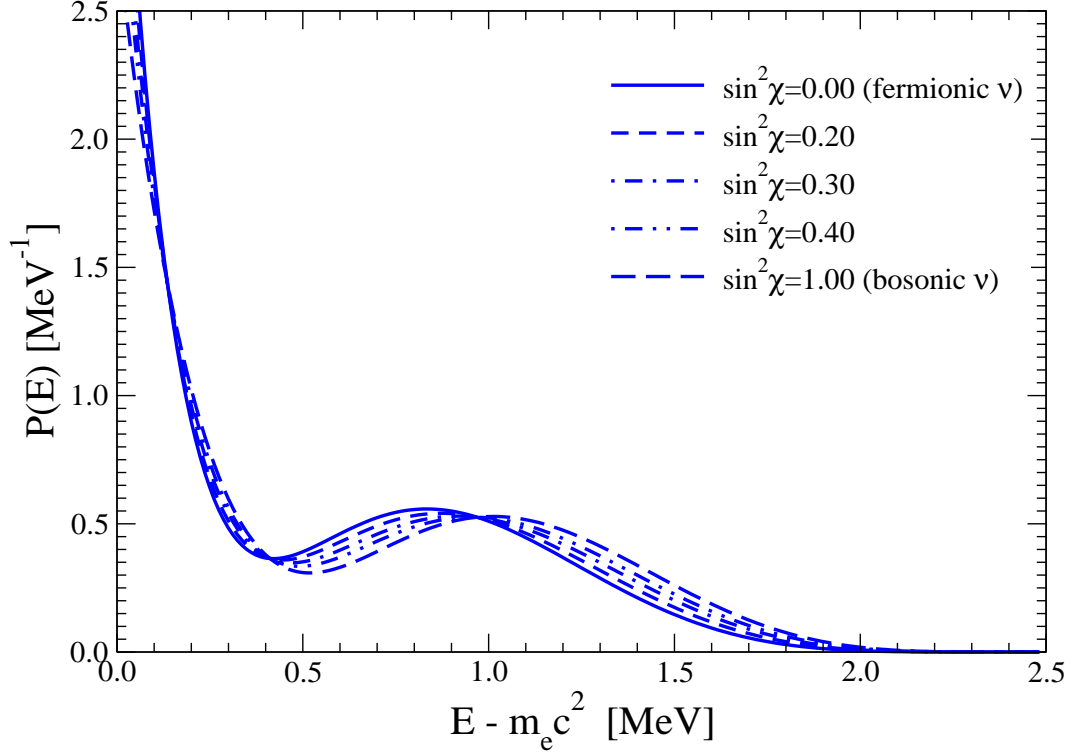


Figure 6: The single electron differential decay rate normalized to the total decay rate vs. the electron energy for $2\nu\beta\beta$ -decay of ^{100}Mo to the excited 2_1^+ state of final nucleus. The results are presented for different values of the squared admixture $\sin^2\chi$ of the bosonic component. The spectra have been calculated in the SSD approximation. The conventions are the same as in Fig. 2.

In the HSD approximation, evaluating the phase space integrals and nuclear matrix elements within the proton-neutron QRPA we find

$$r_0(0_{g.s.}^+) = 0.0014. \quad (47)$$

This smallness is related to a large extend to high energies of the intermediate states, $E_m - E_i$ in comparison with leptonic energies restricted by the energy release $E_l < (E_i - E_f)/2$: $E_l \ll E_m - E_i$. According to (9) the factors K_m^b , L_m^b and consequently the rate are zero in the limit $E_l = 0$. In the lowest approximation we obtain

$$K_m^b, L_m^b \sim \frac{[(E_{\nu 2} - E_{\nu 1}) \pm (E_{e 2} - E_{e 1})]}{(E_m - E_i)^2}, \quad (48)$$

(where plus sign is for K -factors). Then the ratio of the rates can be estimated as

$$r_0(0_{g.s.}^+) \sim \frac{\epsilon_l^2}{4(E_m - E_i)^2}, \quad (49)$$

where ϵ_l is the average energy of the lepton. Taking parameters of the ^{76}Ge -decay we find $r_0 \approx 10^{-3}$ in a good agreement with the calculations in QRPA.

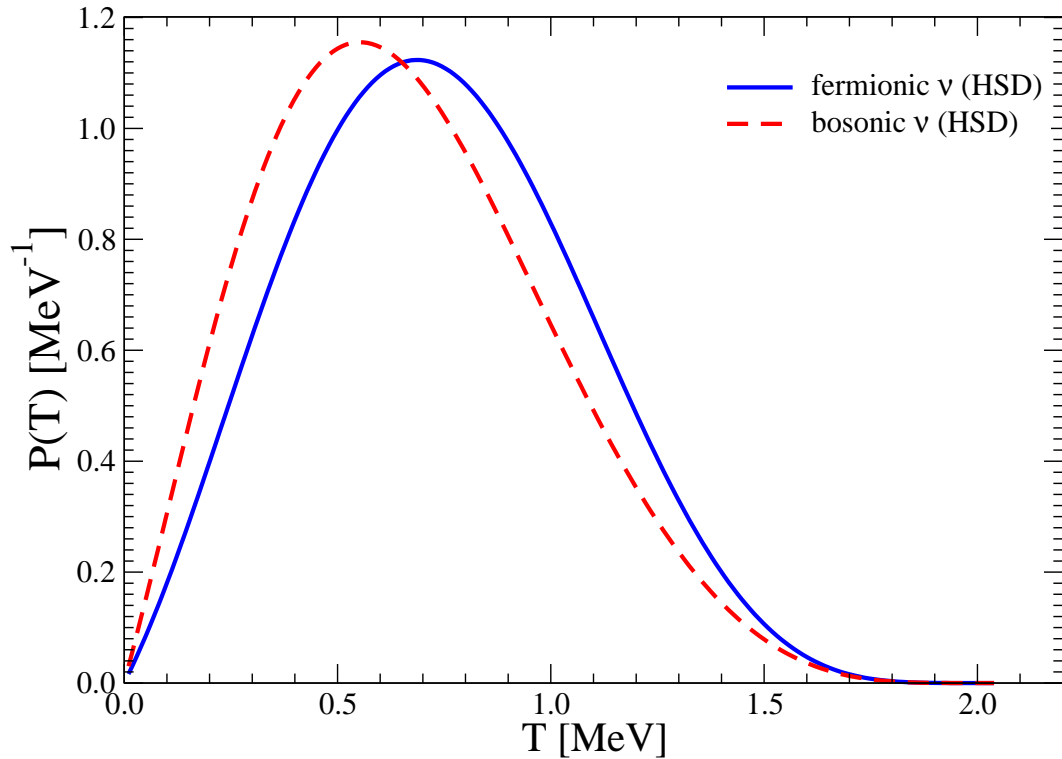


Figure 7: The differential decay rates normalized to the total decay rate vs. the sum of the kinetic energy of outgoing electrons T for $2\nu\beta\beta$ -decay of ^{76}Ge to the ground state of final nucleus. The results are presented for the cases of pure fermionic and pure bosonic neutrinos. The calculations have been performed with the HSD assumption.

In Fig. 7 we show the normalized distributions of the total energy of two electrons for pure fermionic and bosonic neutrinos. As in the case of ^{100}Mo , the decay with bosonic neutrinos has softer spectrum. The energy distribution of single electron is shown in Fig. 8

Due to a small value of $r_0(0_{g.s.}^+)$ a substantial effect of the bosonic component should show up only for $\sin^2 \chi$ being very close to 1: $(1 - \sin^2 \chi)^2 \sim 10r_0(0_{g.s.}^+)$. So studies of the spectra are not sensitive to $\sin^2 \chi$. In contrast, the total rate of the ^{76}Ge decay gives a strong bound on $\sin^2 \chi$.

5 Bounds on bosonic neutrinos

One can search for/restrict the bosonic or partly bosonic neutrino using total rates, ratios of rates of the transitions to the excited and ground states, energy spectra, and angular distributions. Let us evaluate the bounds on $\sin^2 \chi$ that can be obtained from the existing data using these methods.

As follows from our general discussion in sec. 3, for $0^+ \rightarrow 0^+$ transitions: $r_0 \ll 1$. For nuclei with small r_0 the best bound on bosonic neutrino fraction can be obtained from the total rates. A modification of the spectrum due to presence of bosonic component is small. In contrast, the strongest modification of the spectrum is expected for the nuclei

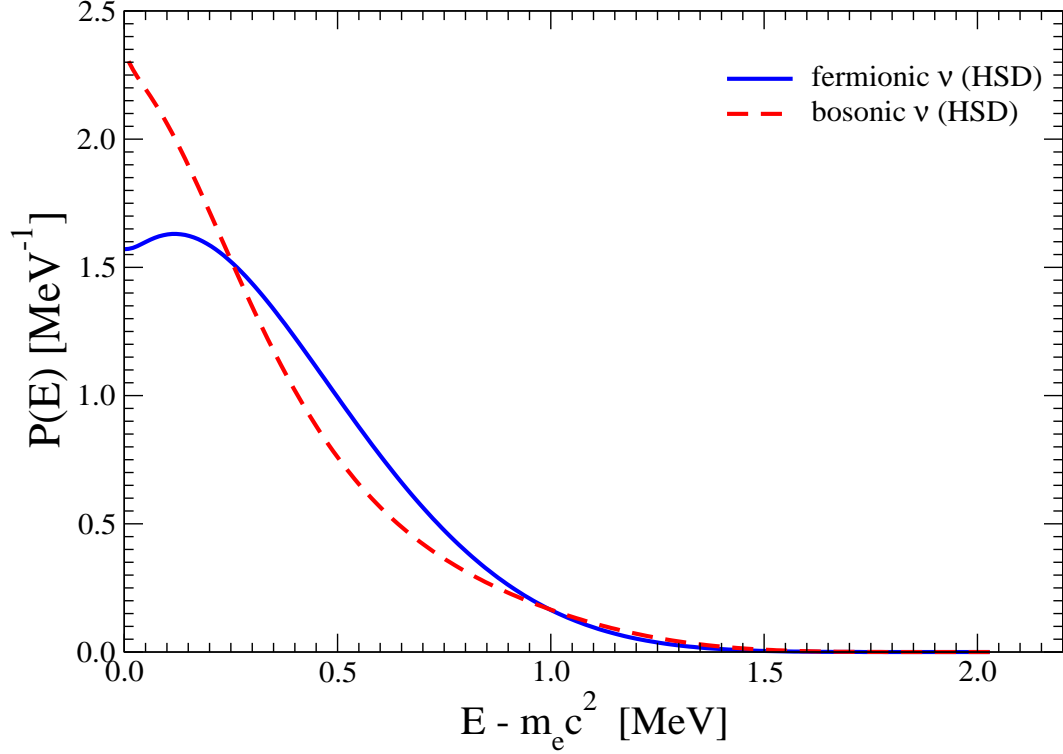


Figure 8: The single electron differential decay rate normalized to the total decay rate vs. the electron energy for $2\nu\beta\beta$ -decay of ^{76}Ge to the ground state of final nucleus. E and m_e represent the energy and mass of the electron, respectively. The results are presented for the cases of pure fermionic and pure bosonic neutrinos. The calculations have been performed with the HSD assumption. The conventions are the same as in Fig. 2.

with large r_0 . This is true, e.g., for $0^+ \rightarrow 2^+$ transition, where $r_0 \gg 1$.

1) *Method 1*: Comparison of the predicted and measured half-life times. Using (15) we can write

$$\sin^2 \chi = \frac{1}{1 + r_0} \left[1 - \sqrt{\frac{T_{1/2}^f}{T_{1/2}^{exp}} - r_0 \left(1 - \frac{T_{1/2}^f}{T_{1/2}^{exp}} \right)} \right], \quad (50)$$

where $r_0 = T_{1/2}^f/T_{1/2}^b$, $T_{1/2}^f$ ($T_{1/2}^b$) are the theoretically predicted life-times for fermionic (bosonic) neutrinos and $T_{1/2}^{exp}$ is the experimentally measured life-time. In the case of agreement between the measured and the predicted (for fermionic neutrinos) life-times, we can use (50) to establish the bound on parameter $\sin^2 \chi$:

$$\sin^2 \chi < \frac{1}{1 + r_0} \left[1 - \sqrt{\frac{T_{1/2}^{f-min}}{T_{1/2}^{exp-max}} - r_0 \left(1 - \frac{T_{1/2}^{f-min}}{T_{1/2}^{exp-max}} \right)} \right]. \quad (51)$$

Here, $T_{1/2}^{f-min}$ and $T_{1/2}^{exp-max}$ are, respectively, minimal theoretical value within a considered nuclear model (e.g., QRPA and its modification, NSM) and maximal experimental value

of the permitted experimental range of the $2\nu\beta\beta$ -decay half-life. For $r_0 \ll 1$ and r_0 smaller the relative accuracy of determination of $T_{1/2}^f/T_{1/2}^{exp}$ the terms proportional to r_0 in (51) can be omitted. Then we get $\sin^2 \chi < (1 - \sqrt{T_{1/2}^{f-min}/T_{1/2}^{exp-max}})$.

Apparently, this method requires knowledge of the nuclear matrix element, and as we mentioned above, reliable estimations can be done for some nuclei e.g., ^{100}Mo and ^{116}Cd assuming SSD hypothesis. For some other nuclear systems nuclear models have to be considered. The two basic approaches used so far for the evaluation of the double beta decay matrix elements are the QRPA and the NSM. For the $2\nu\beta\beta$ -decay of ^{76}Ge the predicted half-lives are $7.7 \cdot 10^{20} - 1.4 \cdot 10^{21}$ years (QRPA) [22] and $1.15 \cdot 10^{21}$ years (NSM) [23]. The experimental half-life (average half-life value is $(1.5 \pm 0.1) \cdot 10^{21}$ years [24]) is in rather good agreement with the theoretical ones for fermionic neutrino within uncertainty characterized by the factor ~ 2 (see [22]). For pure bosonic neutrinos $r_0(0_{g.s.}^+) \approx 10^{-3}$ (QRPA) and therefore for the half-life time we would have $T_{1/2}^b \approx 1.5 \cdot 10^{24}$ years, which is in contradiction with the experimental value. So, purely bosonic neutrino is certainly excluded.

The axial-vector coupling constant g_A is a significant source of uncertainty in the theoretical calculation of the $2\nu\beta\beta$ -decay rate, which is proportional to g_A^4 . The commonly adopted values are $g_A = 1.0$ (by assuming quenching in nuclear medium) and $g_A = 1.25$ (as for free nucleon). This gives about 1.5 uncertainty in NME's.

For factor 2 uncertainty in NME we obtain factor 4 uncertainty in $T_{1/2}^f$. Therefore taking $T_{1/2}^f \sim T_{1/2}^{exp}$, we can put the bound

$$\frac{T_{1/2}^{f-min}}{T_{1/2}^{exp-max}} > \frac{1}{4}. \quad (52)$$

Then, eq. (51) gives

$$\sin^2 \chi < 0.50. \quad (53)$$

Notice that uncertainty in $T_{1/2}^f$ (and not r_0) dominates in this bound.

We can also use the half-life time of ^{100}Mo . Here $r_0(0_{g.s.}^+)$ is much larger (41) but the accuracy of calculations of NME is better. Taking SSD approximation we can calculate the half life with 50% accuracy: $T_{1/2}^f = (6.84 \pm 3.42) \cdot 10^{18}$ years [14]. This value is in agreement with NEMO-3 value, $T_{1/2}^{exp} = (7.11 \pm 0.54) \cdot 10^{18}$ years [16]. Plugging these numbers into (51) we obtain for $r_0(0_{g.s.}^+) = 0.086$

$$\sin^2 \chi < 0.34. \quad (54)$$

Notice that the accuracy of predicted half-life value is connected with experimental accuracy for EC (electron capture) half-life of ^{100}Tc [25]. This accuracy can be improved in the future experiments² down to $\sim 10\%$ and correspondingly, the sensitivity to $\sin^2 \chi$

²In ref. [25] Mo enriched to 97.4% was used and the main background was connected with X-rays from different Tc isotopes which were produced in the sample due to (p,n) and (p, α) reactions on different Mo isotopes, from ^{92}Mo to ^{98}Mo ; see Table II in [25]. If one uses Mo enriched to 99% (or more) then the mentioned above background would be much lower and the accuracy of the measurement would be several times better.

can reach ~ 0.1 . Unfortunately, there is only one (not very precise) EC measurement for ^{100}Tc and thus the above limit on $\sin^2\chi$ is not reliable enough.

Even stronger bound can be obtained from studies of ^{116}Cd -decay. Recently a precise estimation of half-life value based on the SSD approximation and information from the $^{116}\text{Cd}(\text{p}, \text{n})$ reaction was obtained: $T_{1/2}^f = (2.76 \pm 0.12) 10^{19}$ years [26]. This prediction is in a very good agreement with experimental value (The experimental average is $(2.8 \pm 0.2) 10^{19}$ years [24]). Using these results we obtain from (51)

$$\sin^2\chi < 0.06. \quad (55)$$

It should be noticed that the result of ref. [26] substantially differs from the earlier estimation $T_{1/2}^f = (1.1 \pm 0.3) 10^{19}$ years [14] (also based on SSD and measured value of electron capture rate of ^{116}In [27]). This result disagrees with the experimental value and could be interpreted as the effect of partly bosonic neutrino with $\sin^2\chi \sim 0.4$.

2) *Method 2:* Measurements of the differential characteristics of the decays: shapes of the energy spectra (sum energy and single electron energy) and angular distribution. Such information is provided now by NEMO-3 for ^{100}Mo , ^{82}Se , ^{116}Cd , ^{150}Nd , ^{96}Zr and ^{48}Ca . In the future the results for ^{130}Te will be also available [16, 18, 19, 28]. In this method one compares the experimental and theoretical energy spectra as well as the angular distribution. In practice one should perform the statistical fit of the spectra by a general distribution (16) with $\sin^2\chi$ being a free parameter. As we have seen the spectral method has substantial sensitivity to $\sin^2\chi$ for nuclei and transitions with large r_0 . That includes ^{100}Mo , as well as transitions to the excited states. ^{76}Ge with very small r_0 has no high sensitivity.

a) Let us consider first the energy spectra of $0_{g.s.}^+ \rightarrow 0_{g.s.}^+$ decay of ^{100}Mo [16]. In the present paper we will not perform detailed statistical analysis of the spectra, postponing this to the time when measurements will be finished and all careful calibrations will be done. Instead, we give some qualitative estimates. There is a reasonable agreement between the predicted energy spectrum of two electrons and the experimental points. Therefore we can certainly exclude the pure bosonic case ($\sin^2\chi = 1$). Furthermore, comparing the results of Fig. 3 (essentially, the relative shift of the maximum of spectrum) with the experimental spectrum we can put the conservative bound $\sin^2\chi < 0.6$. In fact, there is no ideal agreement between data and theoretical spectrum. A better fit can be obtained for $\sin^2\chi \sim 0.4 - 0.5$.

b) Let us comment on the single-electron energy spectrum from ^{100}Mo decay. The data reasonably well agree with the predictions from the fermionic SSD mechanism, but some difference exists between the data and the fermionic HSD-mechanism predictions. From this it was concluded that the SSD mechanism is more relevant here [18, 19]. Comparing the experimental data and spectra for partly bosonic neutrinos (Fig. 4) we obtain: $\sin^2\chi < 0.7$.

Notice that the SSD spectrum does not show an ideal agreement with data either. There is some discrepancy, especially in the low energy region ($E = 0.2 - 0.4$ MeV). That could be explained by the effect of partly bosonic neutrinos with $\sin^2\chi \sim 0.5 - 0.6$.

Complete analysis of all existing NEMO-3 information (energy and angular distributions) using e.g. maximal likelihood methods, will have a higher sensitivity to $\sin^2 \chi$. However, it is difficult to expect a better bound than $\sin^2 \chi \sim 0.4 - 0.5$, mainly because of the existing disagreement between the data and Monte Carlo (MC) simulations. In fact, it can be just some systematic effect connected to the present poor understanding of response function of the detector. If in future the NEMO experimental data turn out to be in much better agreement with the MC-simulated spectrum, the sensitivity to partly bosonic neutrino will be improved down to $\sin^2 \chi = 0.2 - 0.3$.

3) *Method 3*: Determination of the ratios of half-lives to excited and ground state,

$$r_{f,b}^*(J^\pi) \equiv \frac{T_{1/2}^{f,b}(J^\pi)}{T_{1/2}^{f,b}(0_{g.s.}^+)}, \quad (56)$$

separately for fermionic and bosonic neutrinos. For $2\nu\beta\beta$ -decay of ^{100}Mo the ratio can be calculated rather reliably using the SSD-approximation. The advantage of this quantity is that the EC amplitude, $[(A,Z) \rightarrow (A,Z+1)$ transition], which is not well determined, cancels in the ratio (56).

For ^{100}Mo the transitions to the ground $0_{g.s.}^+$ and excited 0_1^+ states were detected, and in fact, some discrepancy has been observed. The corresponding experimental ratio r^* equals

$$r_{exp.}^*(0_1^+) \simeq 80 \quad (57)$$

(NEMO-3 results [16, 21]), whereas within the SSD approach the calculated ones are

$$\begin{aligned} r^*(0_1^+) &\simeq 61 && \text{(fermionic } \nu) \\ &\simeq 73 && \text{(bosonic } \nu). \end{aligned} \quad (58)$$

A bosonic neutrino fits the data slightly better but the differences are probably beyond the accuracy of the SSD assumption. Still it is also necessary to improve statistics in measurements of the transition to excited 0_1^+ state.

Contrary to the case of 0^+ excited state, the ratio of $2\nu\beta\beta$ -decay half-lives to excited 2^+ and ground state is expected to be strongly different for bosonic and fermionic neutrinos. Using the SSD approximation for the $2\nu\beta\beta$ -decay of ^{100}Mo we found

$$\begin{aligned} r^*(2_1^+) &\simeq 2.5 \cdot 10^4 && \text{(fermionic } \nu) \\ &\simeq 2.7 \cdot 10^2 && \text{(bosonic } \nu). \end{aligned} \quad (59)$$

The $2\nu\beta\beta$ -decay of ^{100}Mo to excited 2_1^+ state has been not measured yet. Using the best experimental limit on the half-life found in [20] we get

$$r_{exp.}^*(2_1^+) > 2.2 \cdot 10^2. \quad (60)$$

This bound is close to the bosonic prediction. A further experimental progress in measuring this nuclear transition will allow one to analyze also the case of partially bosonic neutrino, and therefore is highly required.

6 Conclusions

A study of the double beta decay can provide a sensitive test of the Pauli exclusion principle and statistics of neutrinos. (Notice, that relation between the statistics of neutrinos and possible (small) violation of the Pauli principle is an open issue.) Appearance of the bosonic component in the neutrino states changes substantially the total rates of the decays as well as the energy and angular distributions. We find, in particular, that the ratio $r_0(0_{g.s.}^+)$ of the rates to ground state for bosonic and fermionic neutrinos, is $< 10^{-3}$ for ^{76}Ge and 0.076 for ^{100}Mo , which excludes pure bosonic neutrinos. For transitions to 2^+ excited states $r_0(2^+) \gg 1$, in particular $r_0(2_1^+) \simeq 7$. However, this $2\nu\beta\beta$ -decay channel has been not measured yet.

We have introduced phenomenological parameter $\sin^2 \chi$ that describes the mixed statistics case of partly bosonic neutrinos. The dependence of the energy spectra and angular correlation of electrons on $\sin^2 \chi$ has been studied. The bound on $\sin^2 \chi$ can be obtained by comparison of the predicted and measured total rates of the decays. In spite of the big difference of the rates for fermionic and bosonic neutrinos, this method does not give strong and very reliable bound on $\sin^2 \chi$ due to uncertainties in NME's. The conservative upper bound $\sin^2 \chi < 0.5$ is found using the ^{100}Mo and ^{76}Ge results. Much stronger bound, $\sin^2 \chi < 0.06$, is obtained from recent studies of ^{116}Cd , however this bound requires further checks.

The method based on the study of the normalized energy and angular spectra is less affected by uncertainties in the NME's. The transitions with large $r_0(J^\pi)$ have the highest sensitivity to spectrum distortions and therefore $\sin^2 \chi$. Using the data on the $0_{g.s.}^+ \rightarrow 0_{g.s.}^+$ transition of ^{100}Mo we obtain the bound $\sin^2 \chi < 0.6$. In the future this bound can be improved down to $\sin^2 \chi \sim 0.2$. The $0_{g.s.}^+ \rightarrow 2_1^+$ transition with $r_0(2_1^+) \simeq 7$ can give much stronger bound, but here new, more sensitive experimental results are needed. We find that modification of the energy spectra due the presence of the bosonic components is opposite for $0_{g.s.}^+ \rightarrow 0_{g.s.}^+$ and $0_{g.s.}^+ \rightarrow 2_1^+$ transitions: for $0_{g.s.}^+ \rightarrow 0_{g.s.}^+$ the bosonic component leads to softer spectrum whereas for $0_{g.s.}^+ \rightarrow 2_1^+$ transitions to harder spectrum of electrons. Also the presence of bosonic component leads to flatter angular ($\cos \theta$) distribution.

Strong bound (potentially down to $\sin^2 \chi \sim 0.1 - 0.05$) might be obtained from measurements of ratios of the decay rates to the 2_1^+ excited and ground state. However, this requires further experimental progress.

We note that currently there are no restrictions on the admixture of bosonic component from the BBN. However, as it was indicated in [7] the future BBN studies will be able to constrain the fermi-bose parameter to $\kappa > 0.5$. The bound on parameter $\sin^2 \chi < 0.6$ from the $2\nu\beta\beta$ -decay results in $\kappa > -0.2$.

In conclusion, the present data allow to put the conservative upper bound on the admixture of the bosonic component $\sin^2 \chi < 0.6$. With the presently operating experiments this bound might be improved down to 0.2. In future one order of magnitude improvement

seems feasible.

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